Estimating Eccentricity of Planetary and Stellar Cores

Tom J. Chalko

Head of Geophysics Division, Scientific Engineering Research P/L, Mt Best, Australia, http://sci-e-research.com

Abstract. Eccentricity of solid cores suspended in liquid planetary/stellar interiors has never been considered possible, because there seemed to be no theoretical basis for such a consideration. This article presents an analysis of gravity-buoyancy equilibrium of a solid core in a spherically symmetric pressure gradient of a spinning planet/star. Elementary mechanics suggests that if a solid core exists - it has to be eccentric. The eccentricity of the Earth's core is estimated on the basis of the generally accepted Earth data. Results suggest that what is currently interpreted as a "spinning inner core anisotropy" can actually be caused by the eccentric core, phase locked to the position of the Moon.

Introduction

A generation of researchers educated within the frames of a given set of fundamental concepts always hesitates to re-evaluate these concepts, because the perspective of giving them up leads to unpleasant situations. No one likes to be proven wrong. Hence, conservatism in science is unavoidable. On the other hand, history of humanity proves that understanding cannot be undone. Improved understanding of Reality of the Universe always prevails, even when it is initially suppressed or ignored.

It is generally accepted that the planetary/stellar density distribution is determined by gravity (Newton's law of gravity). Since the minimum of gravitational potential corresponds to spherically symmetric mass distribution, such a distribution seems to be universally accepted in planetary science and cosmology as the only possible distribution.

It is also generally accepted, that the reality of the planetary/stellar interior is determined by the equilibrium between gravity and molecular/atomic forces that resist gravitational compression. This equilibrium manifests itself as a hydrostatic pressure distribution in the planetary/stellar interior.

The presence of a solid object (an object whose deflections are small in comparison to its size) submerged in the liquid planetary interior introduces additional conditions. Behavior of such an object depends not only on the gravitational attraction to the remainder of the planet/star, but also on the buoyancy induced by the hydrostatic pressure gradient in a planetary/stellar interior.

It is demonstrated, that when the effect of buoyancy is admitted for consideration, other than concentric positions of solid cores satisfy the fundamental laws of mechanics in spinning planets and stars.

Determining buoyancy

The buoyancy force that acts on a submerged solid is defined as a resultant (a vector sum) of all hydrostatic pressure forces that act on the surface of that solid. For objects of arbitrary shape the evaluation of buoyancy force is most convenient using vector integral calculus and the divergence theorem of Gauss *[7]*. Imagine a solid object of volume V submerged in the fluid with the arbitrary pressure distribution $p(x, y, z)$ in the inertial Cartesian frame of reference defined by unit vectors $(\mathbf{i}, \mathbf{j}, \mathbf{k})$. The component F_x of the buoyancy force along the x axis is a sum (integral) of all pressure forces $-\mathbf{n}\,p$ that act on the surface S of the object, projected onto the x axis (the $\mathbf{i}$ direction): $F_x = \iint_S \mathbf{i}\cdot(-\mathbf{n}\,p)\;da = -\iint_S (\mathbf{i}\,p)\cdot\mathbf{n}\,da \stackrel{Gauss}{=} -\iiint_V \boldsymbol{\nabla}\cdot(\mathbf{i}\,p)\;dV = -\iiint_V \frac{\partial p}{\partial x}\,dV$,
where the vector $\mathbf{n}$ is the outer unit normal vector of S (pointing to the outside of S hence the minus in $-\mathbf{n}\,p$) and $\boldsymbol{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$.
After determining the remaining components F_y and F_z in a similar way we have:

$$\mathbf{F} = \mathbf{i}F_x + \mathbf{j}F_y + \mathbf{k}F_z = -\iiint_V \boldsymbol{\nabla} p\, dV \qquad (1)$$

Equation (1) clearly demonstrates that the Archimedes principle (that is typically used to estimate buoyancy on the basis of density of the solid and the density of the fluid) is valid only when the pressure gradient in the fluid is *uni-directional* and can be reduced to the constant of the form $\boldsymbol{\nabla} p = \rho\mathbf{g}$, where ρ is the average density of the fluid displaced by the submerged object and $\mathbf{g}$ is the gravity acceleration vector in the direction of the free fall of the object. Only then the buoyancy force $\mathbf{F}$ becomes equal and opposite to the weight of the displaced fluid $\mathbf{W} = \rho\mathbf{g}V$ as stated by Archimedes.

In any other situation the Archimedes principle is simply not valid. Specifically, for a solid object surrounded by near spherically symmetric pressure gradients (such as a planetary or stellar nucleus suspended in planetary/stellar interior) the estimation of the buoyancy force must include explicit integration of all pressure forces that act on the entire submerged surface of that object.

For a near-concentric spherical core submerged in a spherically symmetric pressure gradient the magnitude of the buoyancy force F_P is (see Appendix E1 for derivation) $F_P = -\frac{16}{45}\pi^2 R^4 G\rho_c\rho_F\left(5\frac{D}{R} - \frac{D^3}{R^3}\right)$, where $D \geq 0$ is the eccentricity, R is the radius, ρ_c is the average density of the core, ρ_F is the average density of the fluid that surrounds the core and $G = 6.67\times10^{-11}\; m^3kg^{-1}s^{-2}$. The negative sign indicates that the buoyancy force F_P pushes the sphere away from the maximum pressure point at $D = 0$ for any $D > 0$.

Hydro-gravitational suspension of planetary/stellar cores

Gravitational attraction provides the restoring force that acts to return the eccentric core to its concentric position. The magnitude of the gravitational attraction F_G is (see Appendix E2 for derivation) $F_G = \frac{16}{9}\pi^2 R^3 G\rho_c\rho_F D$. For the purpose of assessing static stability of the concentric equilibrium position of the core it is not necessary to consider hydrodynamic and self-induced electrodynamic forces, because for them to exist the core must be eccentric and move in the planetary interior. Hydrodynamic and self-induced electrodynamic forces will dissipate the core kinetic energy and reduce its eccentricity.

The elastic properties of the hydro-gravitational suspension of the concentric core can therefore be determined by the sum of two forces $F_{HG} = F_G + F_P$

$$F_{HG} = \frac{16}{45}\pi^2 RG\rho_c\rho_F D^3 \tag{2}$$

At first glance the total hydro-gravitational force F_{HG} is positive for any value of eccentricity $D > 0$, indicating that the hydro-gravitational suspension of the core generates a force that returns the core to its concentric position. However the stiffness of this suspension $\frac{dF_{HG}}{dD}$ (the restoring force dF_{HG} for infinitessimally small eccentricity dD) is zero at $D = 0$. This indicates that the concentric equilibrium of the core at $D = 0$ is a neutral (neither stable nor unstable) equilibrium. Let's try to explore what happens to this equilibrium when a planet/star spins.

Core eccentricity in spinning planets/stars

Consider two masses m_1 and m_2 that are D distance apart and spin around one another in a steady state rotation with angular velocity Ω.The elastic properties of the hydro-gravitational connection between these two masses can be represented by a force between them of the form $F_{HG}(D) = AD^3$, where $A = \frac{16}{45}\pi^2 RG\rho_c\rho_F$ can be considered to be constant.

According to laws of classical mechanics, in steady-state rotation, the force in the elastic suspension $F_{HG}(D)$ should be equal to centrifugal inertia forces. Assuming that the entire system spins around its center of mass that can be considered to be an origin of an inertial frame of reference, we have

$$\frac{m_1 m_2}{m_1 + m_2}\Omega^2 D = AD^3 \tag{3}$$

The trivial solution ($D = 0$) represents an unstable configuration that cannot be sustained in a real system. The non-trivial solution of (3) for the eccentricity D is $D = \frac{\sqrt{A(m_1+m_2)m_1 m_2}}{A(m_1+m_2)}\Omega$. Taking into account that the total mass of the planet/star is $M = m_1 + m_2$, and the mass of the core $m_1 = \rho_c \frac{4}{3}\pi R^3$, the non-trivial solution becomes

$$D \approx \frac{\Omega R}{2}\sqrt{\frac{5\,(3M - 4\pi\rho_c R^3)}{\pi G M \rho_F}} \tag{4}$$

According to the expression (4), a planetary/stellar core can be concentric ($D = 0$) only if one of the following three conditions is satisfied: 1) the planet/star doesn't spin ($\Omega = 0$) or 2) the core radius is zero ($R = 0$) or 3) the core mass is equal to the total mass M of the planet/star. None of these conditions is realistic. Hence, it seems that *if a solid core exists in a planetary/stellar liquid interior - it must be eccentric*.

Even though the expression (4) was derived for the simplest possible centrifugal model in an inertial frame of reference and its accuracy may be limited due to simplifications made in the estimation of the hydro-gravitational suspension properties of the core, the validity of the above conclusion appears to depend only on the validity of laws of classical mechanics (Newton's laws of gravitation, motion and equilibrium of forces). Hence - eccentric cores should be very common in spinning planetary and stellar objects. Let's try to find some local evidence.

Eccentricity of Earth's core

For the generally accepted Earth data (PREM *[1]*) ($M = 5.974 \times 10^{24}$ kg, $\Omega = 2\pi/(24 \times 60 \times 60 \times 27.3)$ rad/s, $R = 1220000$ m, $\rho_c = 13000\,\mathrm{kg/m^3}$, $\rho_F = 12000\,\mathrm{kg/m^3}$) expression (4) gives $D = 3935$ [m], which is about 0.3% of the radius of the core R. Hydrodynamic and tidal forces that have been disregarded in the above analysis will modify the core eccentricity. The actual value of D can be substantially different than 3935 [m].

It is interesting to note, that since the density of the fluid surrounding the core is only 8% smaller than the density of the core *[1]*, the shift of the planetary center of gravity caused by the core eccentricity of 3935 [m] will be only $\frac{4}{3}\pi R^3(\rho_c - \rho_F)D/M = 5$ [m]. A few metre shift of the planetary center of gravity due to core eccentricity can easily be attributed to other factors, like a shape of a geoid or a tidal bulge for example, and hence go unnoticed.

Evidence from seismology

An eccentric core becomes the subject of gravitational attraction of other bodies in the Solar system. The attraction from the Moon is dominant, because of its proximity. For this reason, the direction of the core eccentricity should be phase locked to the current position of the Moon. The contribution from the Sun and other significant masses in the Solar system should be observable, but smaller. Hydrodynamic and electromagnetic forces induced by the core moving in the surrounding liquid in order to follow the Moon, should cause a phase delay between the position of the core in the planetary interior and the position of the Moon with respect to Earth.

To a seismologist, who performs measurements with respect to the surface of Earth and tries to reconstruct parameters of the core *[2]*, the ”apparent anisotropy” of the Earth's core should appear to ”rotate” inside Earth. Since the eccentricity of the core must be phase locked to the position of the Moon, the fundamental harmonic Ω_{core} of this apparent rotation observed by a seismologist is simply $\Omega_{core} = \Omega + \Omega_M \approx \Omega + \frac{1}{27.3}\Omega \approx 1.037\Omega$, where $\Omega_M \approx \frac{1}{27.3}\Omega$ is the relative angular velocity of the Moon with respect to Earth. In other words, if the Earth's core is ecentric, the ”apparent anisotropy” of the core identified from seismic measurements should make one ”turn” with respect to Earth's surface in about 27.3 days, exactly as the Moon does. The "anisotropy" of the core should appear to spin 3.7% faster than Earth. On the basis of seismic measurements Su, Dziewonski and Jeanloz *[2]* estimated this figure to be much lower, but their conclusions are likely to be the result of "aliasing". Aliasing occurs when a process is observed at discrete time intervals that are too long in comparison to the true period of the process. In order to identify the core motion period of 27.3 days without any doubt it would be necessary to have an earthquake of a suitable magnitude and location every 3 days or so. Since large earthquakes do not happen this often (yet), conclusions of Su, Dziewonski and Jeanloz *[2]* are based on aliases, not reality.

Some consequences of core eccentricity

Magnetic field and pole reversals. The origin of Earth's magnetic field remains one of the most important unexplained mysteries in planetary science. There is also no plausible explanation for magnetic pole reversals that are so well recorded in the magnetized mineral deposits around the globe *[6]*. Since observations prove that the Earth's magnetic field originates in the core, an assumption of a concentric core provoked scientists to develop a belief that the core is composed from some ferromagnetic alloy. However, this belief cannot explain magnetic pole shifts followed by long periods of a fairly stable magnetic field.

An electrically charged eccentric core seems to offer a simple and elegant explanation of the origin of planetary/stellar magnetism. Temporal changes in the electrical charge of eccentric cores, seem to explain magnetic pole reversals (”pole shifts”) observed not only in planetary, but also stellar objects such as the Sun. Since eccentric cores need to change their electrical charge in time, it is almost certain that they are composed from slowly changing combination of isotopes, providing the mechanism for change in core composition and charge. For this reason, eccentric planetary and stellar nuclei can be considered nuclear reactors that can generate heat.

On the basis of the above described mechanism of magnetic field generation we can also conclude that:

- planets that have no moons and spin slowly around their axis should have weak magnetic fields
- planets with multiple moons should have complicated magnetic fields that change in time
- planets/moons that are phase locked to their orbiting partners will have small Ω and hence a weak magnetic field.

Interplanetary/interstellar torque exchange. Eccentric cores of planetary/stellar objects nearby provide a mechanism for transfer of the angular momentum (torque) between planets/stars and their satellites. The larger the mass ratio of the orbitting partners, the larger the observable temporal changes in their relative orbits should be.

The currently adopted theory of torque exchange implies "tidal bulges" induced by orbiting partners. This theory however, cannot explain observable spiral trajectories of artificial solar satellites, simply because all objects spinning in the solar system taken together cannot cause any "bulge" of the Sun. The presence of an eccentric solar core, however, explains the spiral trajectories of solar satellites quite well. Admitting eccentricity of the solar core for consideration one can predict that the lighter a solar satellite is (the smaller its inertia) - the steeper its spiral trajectory around the Sun would be. This mechanism suggests that small outer planets that orbit the Sun in the same plane as a larger planet - in time can become its moons.

The evidence of the torque exchange between Earth and Moon obtained from lunar laser ranging measurements *[9]* seems to suggest that both bodies have eccentric nuclei.

Dissipation of the kinetic energy of the spin. Work done by hydrodynamic and electromagnetic forces, induced by eccentric planetary/stellar cores moving in the surrounding liquid in order to follow positions of other planetary/stellar objects nearby, provides a mechanism for dissipation of the kinetic energy of spin. Due to this mechanism planets/stars with eccentric hydro-gravtationally suspended nuclei gradually slow down their rotation and eventually stop spinning independently around their own axes as their rotations become ”phase locked” to the nearest star or other body that they orbit.

Our Moon seems to be in such a situation. Detailed topography of the Moon obtained from the lunar satellite Clementine lidar data in 1997 *[10]* indicates that the center of mass of the moon is indeed eccentric with respect to the moon's outer surface by some 1.9 km. Not surprisingly, this eccentricity is pointed toward Earth - the closest celestial object to the Moon. The presence of a phase angle between the eccentricity of the Moon's center of mass and the direction Moon-Earth indicates that the lunar inner core is still suspended elastically inside the lunar interior. Since the mass of the lunar inner core may be only a small portion of the entire mass of the moon, the eccentricity of the solid inner core is likely to be much larger than 1.9 km.

Our moon is not the only celestial body that stopped spinning independently around its own axis. There are other moons and even planets in our solar system (Pluto) that stopped spinning independently around their own axes and their rotations became ”phase locked” to their orbiting partners. For example Pluto and its moon Charon are both phase-locked to one another.

It is important to note that such phase locking is theoretically impossible for planets and moons with concentric (spherically symmetric) density distributions, simply because it is impossible to apply a torque to such bodies using distant forces of gravitational attraction. The elastic ”phase locking” can only occur if there exists an efficient mechanism for torque transfer and dissipation of the kinetic energy of the spin inside every moon and every planet. Hydro-gravitationally suspended eccentric inner cores provide such a mechanism.

Gravitational anomalies. Since internal positions of eccentric nuclei change according to positions of Earth and Moon with respect to the Sun and other bodies in the Solar system, centers of gravity of both Earth and Moon do not remain stationary in their local (geocentric / selenocentric) frames of reference. Any non-simultaneous gravitational field measurement around Earth would necessarily contain ”unexplainable inconsistencies”, unless variable positions of its eccentric inner nuclus is taken into account. It is very likely that positional variability of Earth’s nucleus can contribute to explaination of notorious irregularities in satellite trajectories observed and reported by NASA and difficulties in mapping "the gravity field" around Earth.

Non-steady convection. Motion of the eccentric core in the planetary/stellar interior and associated hydrodynamic phenomena prevent a steady-state convection to become established in the liquid that surrounds the core.

Changes in planetary/stellar axes of rotation. The eccentricity of the core undergoes changes on a long time scale (Ma) due to changes in the density distribution in the decaying planetary interior and thermal changes in the liquid providing hydrodynamic suspension of the core. Due to the near spherical geometry of Earth, very small changes in the eccentricity of the core may cause dramatic changes in the *orientation* of Earth’s principal axes of inertia, and cause the entire planet to alter its axis of rotation. Since the inner core is suspended elastically, it is almost certain that such a transition involves large angular oscillations of the entire planet before a new axis of rotation becomes established. It is also highly likely that large oscillations of the inner core during such an event induce intensive and global volcanic activity.

As the eccentricity of the inner core gradually changes, it seems inevitable that, from time to time, sudden and very major adjustments to the Earth’s axis of rotation take place.

Explaining anomalies in lunar geology. A long standing, and yet unsolved lunar puzzle is the fact that the ”near” side of the moon, visible from Earth, is structurally very different than the ”far” side of the moon *[10]*. Again, the eccentricity of the lunar inner core reactor provides a plausible explanation. The solid inner core decaying by means of spontaneous nuclear fission is a major source of heat inside the moon. Since the lunar core eccentricity is phase locked to Earth, the ”near” half of the moon receives systematically more heat than the ”far” part. Over time, the temperature differences cause observable differences in the lunar surface appearance between the ”near” and the ”far” sides.

Conclusions

The phase correlation between the apparent ”anisotropy” of the inner core and the position of the Moon should be relatively easy to verify using an updated tomographic model of Earth that admits such a possibility. Note, that tomography can only provide non-unique solutions. Hence, if the tomographic model of the core does not allow certain features, they will never be found. If the core eccentricity is confirmed by tomographic analysis of seismic data, several aspects of planetary sciences and cosmology may need a major revision.

The actual pressure distribution in the planetary interior may differ from the distribution estimated on the basis of hydrostatic compression and spherical symmetry. One of the reasons for such a difference may be the ability of the mantle and the crust to carry a tensile load. According to the presented analysis, even a very slight variation in the pressure gradient around the core may significantly change its buoyancy and hence its eccentricity.

The most serious consequence of the analysis presented in this article is a possibility of the inner core of Earth to be a nuclear fission reactor, rather than some crystallizing solid as it is generally accepted today. Such a reactor generates heat in its entire volume, but its cooling can occur only at its surface. The heat generated in the core can only escape into space via Earth’s atmosphere. Hence, the properties of the atmosphere limit the cooling rate of the planetary interior, including the cooling rate of the core. If the rate of heat generation in the planetary interior is greater than the rate of cooling, even by a tiny amount, the core reactor accumulates heat over time. Overheating the planetary core and the planetary interior would lead to:

1. Accelerated melting of polar ice caps, heated from underneath. This should be one of the first symptoms, because the ratio of the geothermic to solar energy is the greatest under the polar ice. Is it a coincidence, that large Antarctic glaciers melt up to 8 times faster today than just a few years ago?*[11][12]* If any of the Antarctic glaciers slides into the ocean we will observe a significant (and instant) rise in sea level. Global flooding is a real possibility.
2. Systematic increase in the speed of motion of tectonic plates, due to declining viscosity of the overheated planetary interior
3. Systematic increase in global volcanic activity and the number of volcanic explosions. This should be first observed near the equatorial plane, due to the core eccentricity. If sufficiently many volcanoes explode, the dust from their explosions will disperse high into the atmosphere and reflect the solar radiation back to space. Without sunshine the planetary surface will freeze and will remain frozen until the volcanic dust settles down, which may take many years. The above mechanism demonstrates that ice ages

can be considered as a natural response to moderate overheating of the planetary interior.

4. If the under-cooled solid core reactor continues to accumulate heat, despite the above described cooling mechanisms, conditions for its meltdown may occur. The meltdown will begin in the center of the planetary core reactor. Since the core is eccentric and spins, the molten part will be subject to centrifugal forces that will eventually segregate and stratify various radioactive isotopes present in the core according to their density. If the molten area of the core becomes large enough, one of the isotopes may reach the critical mass. In such a case, the geothermic energy that was scheduled to be released over billions of years will be released in a fraction of a second and the planet will explode. Interestingly, historical records reveal the evidence of planetary explosion in our solar system (Appendix E3).

Above conclusions indicate that the greatest danger to our civilization is not a slow climate change, but overheating of the planetary interior caused by polluted atmosphere that traps increasingly more solar heat. Do we have enough integrity and intelligence to comprehend and analyze the danger before it is too late? Our civilization will not be the first one on Earth to vanish, but it can be the last...

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Appendix E1. Buoyancy of spherical object in spherically symmetric pressure gradient

Consider a fluid with spherically symmetric pressure distribution p about point O - the center of an inertial frame of reference. Since the pressure distribution is radially symmetric, without a loss of generality we can orient our coordinate system so that the position of the spherical object away from the maximum pressure point O is measured along the Z axis as in Fig 1.

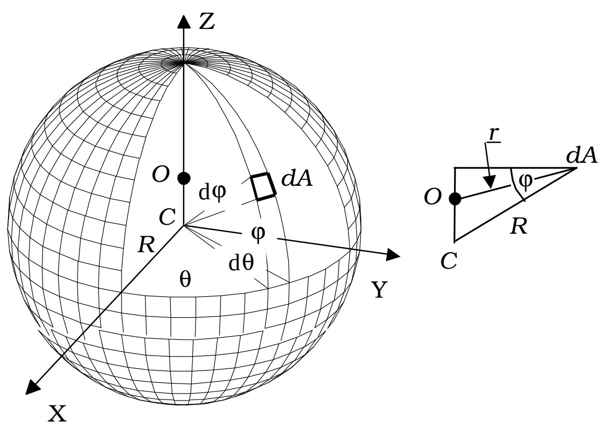

Fig 1. Spherical system of coordinates. $dA = R^2 \cos\varphi d\varphi d\theta,\; r = \sqrt{R^2 + D^2 - 2RD\sin\varphi}$ and $D = \overline{OC}$

The resultant force on a solid spherical object of radius R located in such a fluid is an integral (a vector sum) of all pressure forces that act on all elements dA on its surface

$$\mathbf{F}_P = -\mathbf{k}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2} R^2\, p(r)\sin\varphi\cos\varphi d\varphi d\theta \ , \tag{5}$$

where $\mathbf{k}$ is the unit vector along the Z axis. In a radially symmetric pressure distribution the pressure is a function of the distance r from the point of maximum pressure O. Considering a linear pressure distribution of the form $p(r) = p_0 + \frac{\partial p}{\partial r}\,|r|$ doesn't restrict the generality of our analysis, because any radially symmetric pressure distribution can be linearized in the vicinity of the surface of the sphere, especially when the center of the sphere is near point O. We have $p(r) = p_0 + aR\sqrt{1+z^2-2z\sin\varphi}$ where $a = \left.\frac{\partial p}{\partial r}\right|_{r=R}$ and $z = D/R$. The surface integral (5) evaluated analytically is:

$$\mathbf{F}_P = \mathbf{k}\frac{4\pi R^3}{15}\left.\frac{\partial p}{\partial r}\right|_{r=R} \times \begin{cases} 5z - z^3 & \text{for} \quad |z| \leqslant 1 \\ 5 - z^{-2} & \text{for} \quad |z| > 1 \end{cases} \tag{6}$$

The expression (6) for the "buoyancy" force $\mathbf{F}_P$ that acts on a solid sphere in a spherically symmetric pressure gradient is a non-linear function of $z = D/R$ and doesn't resemble the Archimedes principle. It doesn't even depend on density. How does it relate to the principle of Archimedes?

Imagine a solid sphere much smaller than its distance from the center of the near-spherical planetary/stellar vessel. Under this condition $z >> 1$, the gradient $\left.\frac{\partial p}{\partial r}\right|_{r=D} = -\rho g$ and the equation (6) becomes precisely equivalent to the Archimedes principle. Hence, the Archimedes principle provides a reasonable approximation for the buoyancy force of a solid submerged in a fluid *only when the size of the solid is much smaller than its distance away from the center of the planet/star.* Is this why limitations of the Archimedes principle have been ignored for 22 centuries?

In order to find a reasonable closed form algebraic expression for $\mathbf{F}_P$ let's try to estimate $\left.\frac{\partial p}{\partial r}\right|_{r=R}$ when $D << R$. It is generally accepted that compression inside planetary and stellar interiors can be considered hydrostatic. The pressure inside a planet or a star can be considered to increase with depth h from the surface, according to the relationship: $p(h) = \int_0^h \rho(h)g(h)dh$ where $\rho(h)$ is density and $g(h)$ is the magnitude of the gravity acceleration at depth h. The magnitude of the gravity acceleration g is a known function of the radial distance r measured from the center of the planet/star: $g(r) = \frac{4\pi G}{r^2}\int_0^r \rho(x)x^2dx$ where $G = 6.67\times 10^{-11} m^3kg^{-1}s^{-2}$. When we combine these relationships (noting that depth $h = R_E - r$ and R_E is the radius of a given planet/star) we can express the pressure p inside the planetary/stellar interior as a function of the radial distance r from the center of the planet/star to be $p(r) = -4\pi G\int_{R_E}^r \rho(s)\frac{1}{s^2}\int_0^s \rho(x)x^2dxds$. The expression for the radial pressure gradient is therefore $\frac{\partial p}{\partial r} = -4\pi G\rho(r)\frac{1}{r^2}\int_0^r \rho(x)x^2dx$ and since it is negative, it indicates that the pressure increases with depth for any radial density distribution $\rho(r)$.

At the solid core boundary the density of the fluid is $\rho(R) \approx \rho_F$ and the pressure gradient is $\left.\frac{\partial p}{\partial r}\right|_{r=R} \approx -\frac{4}{3}\pi G\rho_c\rho_F R$, where ρ_c is the average density of the core. Inserting the expression for $\left.\frac{\partial p}{\partial r}\right|_{r=R}$ into (6) gives the buoyancy force for infinitessimally small D.

$$\mathbf{F}_P \approx -\mathbf{k}\frac{16}{45}\pi^2 R^4 G\rho_c\rho_F\left(5\frac{D}{R} - \frac{D^3}{R^3}\right) \tag{7}$$

Appendix E2. Gravity force on the inner core

Consider a solid spherical core of radius R and mass m_c inside a spherically symmetric vessel filled with fluid with a density ρ_F. Denote by D the displacement of the core from the centre of the vessel O - an origin of an inertial frame of reference. The gravitational interaction between the solid core and the liquid in the vessel is determined solely by the gravitational attraction of the liquid contained inside the sphere of radius $R + D$, indicated in Fig 2 as a shaded area. Again, without a loss of generality, we can orient our system of coordinates so that the displacement of the solid core is measured along the Z axis.

Consider an infinitesimally small part dm of the liquid, in the form of a fragment $d\varphi\, d\theta$ of the spherical shell of radius r and thickness dr. The gravity force that will attract the core toward dm is

$$dF_G = \frac{G}{r^2}m_c dm = Gm_c\rho_F dr\cos\varphi d\varphi d\theta, \tag{8}$$

where G is the gravity constant. In order to find the total gravity force that attracts the solid core to the centre of the vessel we need to integrate the gravitational forces dF_G over the entire volume indicated in Fig 2 by the shaded area. Due to the axial symmetry about the Z axis, only the Z components $dF_G\sin\varphi$ will contribute to the total force $\mathbf{F}_G$. Details of the integration are presented below, considering

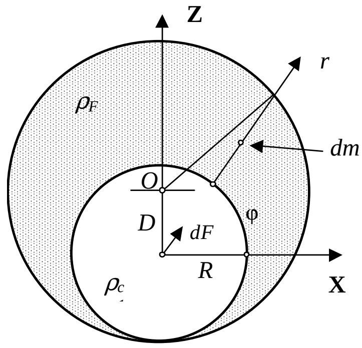

Fig 2. Solid sphere R displaced by D from the centre O of a spherically symmetric liquid. $dm = \rho_F r^2 \cos\varphi d\varphi d\theta dr$

that the mass of the solid core is $m_c = \frac{4}{3}\pi R^3 \rho_c$ and ρ_c is its average density.

$$\begin{aligned}
\mathbf{F}_G &= \mathbf{k} G m_c \rho_F \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \int_R^{D\sin\varphi + \sqrt{D^2\sin^2\varphi + R^2 + 2RD}} dr \sin\varphi \cos\varphi d\varphi d\theta = \\
&= \mathbf{k} G m_c \rho_F \int_0^{2\pi} \int_{-\pi/2}^{\pi/2} \left(\sqrt{D^2 \sin^2\varphi + R^2 + 2RD} + D\sin\varphi - R \right) \sin\varphi \cos\varphi d\varphi d\theta \\
&= \mathbf{k}\frac{4}{3}\pi G m_c \rho_F D = \mathbf{k}\frac{16}{9}\pi^2 R^3 G \rho_c \rho_F D \qquad (9)
\end{aligned}$$

The above result indicates that the magnitude of the gravity force is proportional to the displacement D. Displacement D doesn't need to be "small" in comparison to R, so long as the density ρ_F of the fluid is constant in the integrated volume and the density distribution of the fluid in the remaining part of the vessel remains spherically symmetric.

When the core R occupies the central position $(D = 0)$ the gravitational force $\mathbf{F}_G = 0$ exactly as it was in the case of the pressure force $\mathbf{F}_P$. However, unlike the pressure force $\mathbf{F}_P$, for any non-zero value of D the resultant gravity force $\mathbf{F}_G$ is always oriented toward the center of the vessel O. It means that gravity is the force that helps to stabilize the central equilibrium position of the inner core.

Appendix E3. Can a planet explode?

If a planet can indeed explode, and there was at least one such event somewhere in our Solar system in the distant past, we should be able to find the evidence of it today. This is due to the fact that the debris from the exploded planet would not vanish. Bits and pieces would not only remain, but their collective presence should still mark a trajectory (the orbit around the Sun) of the planet that exploded.

In Greek Mythology there is a story about a planet that exploded. The planet was called Phaëthon. Did our ancestors embed this event in their belief system because they actually witnessed a planetary explosion and they just couldn't explain it any other way? Can we determine **today** what is a myth and what is an actual fact?

It is a well-known fact that there exists the so-called "asteroid belt" in our Solar system. It is a "belt" of a large number of asteroids that orbit the Sun along orbits that are located between Mars and Jupiter. At least 40,000 of these asteroids are thought to have diameters larger than 0.8 km (0.5 mile). The largest asteroid in the asteroid belt, called Ceres, is about 930 kilometers across.

The existence and the origin of the entire asteroid belt are long standing scientific puzzles. Why does the asteroid belt exist only between Mars and Jupiter and there are no asteroid belts between other planets?

The present belief is that planets in the solar system formed out of randomly distributed dust and other bits and pieces. Hence, it is also believed that the growth of a full-sized planet between Mars and Jupiter was "aborted" during the early evolution of the solar system.

The explosion of a planet that existed between Mars and Jupiter is a much more logical and plausible explanation. Plato, one of the greatest writers and philosophers of all time, was aware that the story of Phaëthon *"destroyed by a thunderbolt"* had its origin in a real planetary event. He wrote *[13]* : *"Now this has the form of a myth, but really signifies decline of the bodies moving in the heavens..."*.

The meaning of the word "phaëthon" ($\varphi\alpha\varepsilon\theta\omega\nu$) in ancient Greek is "giving light, luminous, brilliant, shining" *[14]*. Note that words "phaëthon" and "photon" originate from the same root ($\varphi\alpha o\varsigma = \varphi\omega\varsigma$) *[14]*. In the myth, Phaëthon is known as "*the son of Helios*" (the son of the Sun) *[13]*. Doesn't this hint that the planet Phaëthon was one of the brightest objects in the sky at night? Isn't it obvious that a disappearance of such an object would attract the attention of even a casual sky observer? The story of the destruction of Phaëthon "*by a thunderbolt*"*[13]* indicates that our ancestors perceived its explosion to be **as bright as lightning**. Should we ignore a witness report of our ancestors embedded not only in their heritage but also **in their language**?