Temperature distribution in a spherical reactor

Tom Chalko, Head of Geophysics Division, Scientific Engineering Research, Mt Best, Australia, sci-e-research.com

Let's consider a homogenous spherical core reactor cooled from the outside. The differential equation governing the conduction and the heat storage in a solid is

$$\nabla^2 T + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

where t is time, ∇^2 is the Laplace operator in the spherical system of coordinates (r, φ, θ), $T(r, \varphi, \theta, t)$ is the temperature distribution, q is the heat generation rate per unit volume of the reactor, k is thermal conductivity and α is thermal diffusivity of the material of the reactor. In the case of spherical symmetry, the temperature distribution T becomes a function of the radial position r and time t only and the equation (1) becomes simplified as follows:

$$\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{2}{r}\frac{\partial T(r,t)}{\partial r} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T(r,t)}{\partial t} \tag{2}$$

The core is cooled by convection, i.e thermal energy is transferred between the eccentric solid core and the fluid that flows around it when the planet spins. The amount of the convection cooling determines the temperature gradient at the surface of the core $\left[\frac{\partial T(r,t)}{\partial r}\right]_{r=R} = \Delta_T(t)$. This condition, together with the obvious condition that the temperature of the outside surface of the core is $T\left(R,t\right) = T_0(t)$, defines boundary conditions required to solve the equation (2). Although the partial differential equation (2) with the above boundary conditions can be solved analytically, we focus on its steady-state solution $\left(\frac{\partial T(r,t)}{\partial t} = 0\right)$, simply because such a solution is sufficient to illustrate the key point of this article. The exact steady state solution of (2) is:

$$T\left(r\right) = -\frac{1}{6}\frac{q}{k}r^2 + C_1 + \frac{C_2}{r} \tag{3}$$

Constants C_1 and C_2 determined from the boundary conditions are: $C_1 = \frac{qR^2}{2k} + R\Delta_T + T_0$ and $C_2 = -(\frac{qR}{3k} + \Delta_T)R^2$. It is interesting to note that the constant C_2 is zero ($C_2 = 0$) only if the temperature gradient on the surface of the reactor (determined by the convection cooling) is $\Delta_T = -\frac{qR}{3k}$. The tiniest changes to the convection cooling of the reactor, and the corresponding gradient Δ_T, lead to the extreme temperature changes in the center of the spherical reactor ($r = 0$). The larger R - the stronger the effect.

Theoretically the temperature at the center of the core $T\left(0\right)$ can become infinitely large, but only when the reduction in cooling (Δ_T) is maintained indefinitely long. (We have to remember that the expression (3) is a steady-state asymptotic solution of the equation (2)). In reality, the temperature gradient Δ_T fluctuates around the value $\Delta_T = -\frac{qR}{3k}$. When cooling is reduced for whatever reason - the reactor accumulates heat, its temperatures rise and the convection cooling becomes more efficient. This in turn causes changes in the gradient Δ_T and the center of the core reactor cools down. Due to the non-linear (hyperbolic) relationship (3), the self-excited thermal oscillations are maintained. Can a similar process in the solar core can explain fluctuations in the activity of the Sun?

Back on Earth, our results clearly indicate that the slightest reduction in the convection cooling of the core (Δ_T), when maintained for a sufficiently long time, leads to the extreme thermal conditions in the center of the core. The cause-effect relationship is not linear. It is HYPERBOLIC.

Hence, if we do not recognize the problem early enough and continue to contribute to planetary interior overheating by increasing greenhouse emissions that trap more Solar heat - we are likely to cause the meltdown of the inner core reactor and its subsequent explosion. Am I expressing myself clearly enough? Good planets are not easy to find...