Combining accelerations and GPS-Doppler velocities

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Abstract.

This article presents a concept of a method of combining measurements of accelerations and GPS-Doppler velocities to reconstruct parameters of motion of a craft. The method is intended for post-processing of the acquired data.

Measured data

Measurement data includes:

1. accelerations along 3-axes associated with moving DC accelerometer
2. three components of GPS-Doppler velocity (SOG-speed over ground, COG-course over ground, CR-climb rate) of the same point.

The objective of the method is to reconstruct parameters of 3D motion on the basis of all measured data

Pre-processing

Acceleration and GPS-Doppler velocity data should be described along the same time axis.

Preliminary experiments (measuring known motion) can determine the time delay between acceleration and velocity recordings.

Raw acceleration data should be then time-shifted to synchronize them with UTC time stamps reported and recorded by the GPS.

Raw acceleration measurements should be also scaled and corrected for measurement offset. If thermal characteristics of accelerometers are available, they should be used in conjunction of temperature measurements to correct for thermal effects in scaling and offset.

Systems of coordinates

Consider local inertial system of coordinates East-North-Up (*ENU*) at Earth's surface close to where GPS measurements are performed.

Let's express GPS-Doppler measurements (SOG,COG, CR) in *ENU* system of coordinates. We have

$$\begin{bmatrix} v_E \\ v_N \\ v_U \end{bmatrix} = \begin{bmatrix} SOG \sin COG \\ SOG \cos COG \\ CR \end{bmatrix} \qquad (1)$$

Consider the local moving and rotating (non-inertial) system of coordinates associated with accelerometer axes, located at the centre of the seismic mass of the accelerometer.

Accelerometer measures accelerations along 3 axes of this system. Denote axes of this system by *xyz*.

Aligning systems of coordinates

Before accelerations (measured by accelerometer) and velocities (measured by GPS-Doppler technique) can be considered together we need to express 3 components of acceleration and 3 components of GPS-Doppler velocity in aligned systems of coordinates. At any time, alignment of the coordinate system *xyz* in relation to *ENU* can be determined by a direction cosine matrix $R(\phi,\theta,\psi)$ as follows

$$R(\phi,\theta,\psi) = \tag{2}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & \sin\theta \\ -\cos\phi\sin\psi - \sin\theta\cos\psi\sin\phi & \cos\phi\cos\psi - \sin\theta\sin\phi\sin\psi & \cos\theta\sin\phi \\ \sin\phi\sin\psi - \cos\phi\sin\theta\cos\psi & -\cos\psi\sin\phi - \cos\phi\sin\theta\sin\psi & \cos\theta\cos\phi \end{bmatrix}$$

where ϕ,θ and ψ are Euler angles, representing 3 degrees of freedom and ψ is the angle of rotation about U axis in the fixed system of reference.

Alignment of *ENU* in relation to *xyz* can be described by matrix R^{-1}, which can be computed as an inverse or R or computed like R in equation (2) by reversing the order and signs of rotations (using angles $-\phi,-\theta$ and -ψ) as follows. Performing these operations analytically demonstrates that $R^{-1} = R^T$.

Relationship between acceleration components in *xyz* and *ENU* can be expressed as follows

$$\begin{bmatrix} a_E \\ a_N \\ a_U \end{bmatrix} = R^{-1}(\phi,\theta,\psi) \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = R(\phi,\theta,\psi) \begin{bmatrix} a_E \\ a_N \\ a_U \end{bmatrix} \tag{3}$$

Accelerations A_N, A_E, A_U in *ENU* system of coordinates can also be computed from GPS-Doppler velocities $v_N; v_E; v_N$ using digital differentiation. In the absence of measurement errors $A = a$. This means that

$$R^{-1}(\phi,\theta,\psi) \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} A_E \\ A_N \\ A_U + g \end{bmatrix} \tag{4}$$

Term g represents gravity acceleration that DC accelerometer measures in addition to kinematic accelerations. Since accelerations obtained by differentiating GPS-Doppler velocities do not contain g, it must be added here to facilitate correct comparison of accelerations.

The matrix expression (4) represents 3 equations with 3 unknowns ϕ,θ and ψ. At the first glance it seems that the system of equations (5) can serve as a basis for determining the mutual angular alignment of coordinate systems *xyz* and *ENU* on the basis of our measurements.

However, there are few difficulties in using equations (4):

1. The system of equations (4) does not have a unique solution for ϕ,θ and ψ. The system of equations (4) may have an infinite number of solutions when ranges ϕ,θ and ψ are unlimited, and may have several solutions when angles ϕ,θ and ψ are restricted to intervals $(-\pi/2, \pi/2)$. Unique solution of (4) may be available only in limited intervals of ϕ,θ and ψ.

So, more information is needed to determine the mutual angular alingment of *xyz* and *ENU* on the basis of measurements that are available to us.

2. For some combinations of input data parameters there are too many solutions. The most vivid situation occurs when all accelerations $a_x = a_y = a_z = 0.$ When there is no acceleration, there is also no information about coordinate system alignment of accelerometer. Such a situation may be frequent in space travel, but is unlikely to occur on Earth. There are not too many people who are interested to study motion of an object travelling directly up with constant acceleration g or a free fall of an object in absence of air resistance. When any 2 components of accelerations are absent, there is a problem too. Consider for example $a_y = a_z = 0.$ In this situation variable ϕ remains undefined by equations (4), because it analytically disappears from them. Fortunately on Earth's surface there exists gravitational attraction, quantified by constant acceleration $g,$ that is directly measured by accelerometer and contributes to all 3 components of measured accelerations a_x, a_y, a_z in all but a few very special cases. Gravity acceleration serves here as a "marker" that helps us to determine the orientation of the accelerometer with respect to Earth. For this reason all special cases discussed here, although possible, should not occur too often in practical motion measurements on Earth. However, any algorithm that is used to estimate mutual alignment of *xyz* and *ENU* should be ready to handle such cases. One possible strategy may involve adopting ϕ and/or θ and/or ψ values from previous data sample when any of the variables cannot be determined with desirable accuracy.
3. Equations (4) are non-linear and there are no convenient analytical methods for solving them.
4. It is quite awkward to use equations (4) directly to solve for ϕ, θ and ψ. In practice, we can only seek ϕ, θ and ψ for which the norm $\|a - A\|$ is minimized. Minimizing the norm $\|a - A\|$ is equivalent to minimizing the least square error.of differences between vectors a and A.

$$\mathcal{G} = \|a - A\| = \left\| R^{-1}(\phi,\theta,\psi) \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} - \begin{bmatrix} A_E \\ A_N \\ A_U + g \end{bmatrix} \right\| \tag{5}$$

where $R^{-1}(\phi,\theta,\psi) =$

$$= \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi - \sin\theta\cos\psi\sin\phi & \sin\phi\sin\psi - \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi - \sin\theta\sin\phi\sin\psi & -\cos\psi\sin\phi - \cos\phi\sin\theta\sin\psi \\ \sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix}$$

Above difficulties force us to seek numerical methods for seeking solutions for ϕ, θ and ψ. An iterative numerical procedure (such as a gradient method seeking minimal norm $\|a - A\|$ in reference [2]]) can be adopted in post-processing of recorded GPS-Doppler and acceleration data sets. Such an iterative procedure has several limitations and requirements:

1. it requires "initial values" for ϕ, θ and ψ to begin the iteration process. Gradient methods are reliable and efficient only when initial values are reasonably close to the sought solution [2].
2. it can only compute "local" solution for ϕ, θ and ψ, defined by a local minimum of the goal function $\mathcal{G}$ defined by equation (5). It is responsibility of the programmer to make sure that the required global minimum of $\mathcal{G}$ has been found.
3. the goal function $\mathcal{G}$ defined by equation (5) needs to be continuous and differentiable along ϕ, θ and ψ

Requirement 3 above seems satisfied for our problem. Satisfying the limitation 1 seems to be a key requirement for obtaining satisfactory solutions for ϕ, θ and ψ. In essence, if we can determine a reasonable set of "initial values" for ϕ, θ and $\psi,$ we should be able to determine the solution for ϕ, θ and ψ (orientation of *xyz* in relation to *ENU*) using an iterative gradient method such as the method described in reference [2].

Example:

Consider GPS-Doppler and accelerometer instrument mounted firmly on top of a helmet of a sailor-windsurfer. Let *z* axis of the accelerometer point up and *x* axis point forward. Since human head remains predominantly vertical, even when body leans at some angle, we should be able to take advantage of the fact that *z* axis will remain close to *U* axis during all normal sailing manoeuvres, except catapult, crash or acrobatics. This means that angles ϕ, θ should be both small. The angle ψ (rotation about *U* axis) can be determined on the basis of the direction of travel (COG) reported by GPS, because it is very likely that sailor will actually look ahead when sailing. So, if an accelerometer is mounted on a helmet with *z* axis pointing "generally up" and *x* axis pointing "generally forward", we should be able to consider initial values for ϕ, θ and ψ to be $0, 0$ and $(\pi/2 - COG)$ respectively.

Similar estimates for initial values of ϕ, θ and ψ can be considered when GPS-Doppler and accelerometer instrument is mounted firmly on a moving craft. The actual solution for ϕ, θ and ψ can reveal the angular orientation of the craft during motion (yaw, pitch and roll).

Sampling rate considerations

GPS-Doppler velocity data *v* is available at 1Hz sampling frequency, while accelerations *a* are sampled at 50Hz.

Alignment of the coordinate system described in the previous section requires both sets of data to be synchronously available. One possible strategy is as follows:

1. compute average accelerations *a* between two corresponding GPS-Doppler samples using the Fourier reconstruction method described in reference [1]. Acceleration *a* needs to be an average acceleration in the time between GPS-Doppler samples.
2. compute average accelerations *A* from GPS-Doppler velocity components. If more than one GPS-Doppler measurement of *v* is available for any UTC instant, they all should be averaged before being used to compute accelerations. Averaging synchronous and independent samples reduces the measurement error.

Motion reconstruction

Coordinate system alignment using the method above is only possible at 1s time intervals (1Hz sampling rate) due to limitations associated with the available data. Assuming continuity of motion (and more specifically, continuity of accelerometer rotations) it should be possible to estimate coordinate system rotations between 1Hz alignment results, computed using the method presented above.

Any interpolation method used here needs to preserve properties of rotation matrix *R* and the associated inter-dependence between Euler angles ϕ, θ and ψ.

Interpolation should be easier and more realistic when alignment between *xyz* and *ENU* systems of coordinates changes slowly, i.e. the bandwidth of alingment changes is much smaller than the Nyquist frequency of GPS-Doppler velocity data (0.5Hz).

Once the alignment between *xyz* and *ENU* is determined for a selected set of time-domain data, we can begin reconstructing kinematic details of motion that was the subject of measurement.

Using the alignment parameters determined above, components of accelerations *a* measured by accelerometer can be expressed in *ENU* system of coordinates and integrated in time to provide the corresponding velocity components and enable direct comparison with velocity components measured directly using GPS-Doppler.

Combining 1Hz GPS-Doppler velocity components with accelerometer-based 50Hz velocity components should take into account corresponding measurement errors.

Conclusions

The presented method for identification of coordinate system alignment presented here needs to be tested using suitable numerical algorithm, such as the algorithm described in reference [2] for example.

Combining accelerations and GPS-Doppler velocities seems possible, but the simplest way to confirm this seems to implement numerical algorithms and perform tests for specific applications. In other words, more research is needed.

References

[1] T.J.Chalko, Reconstructing data from alias-free discrete series of samples, NU Journal of Discovery, March 2009, http://nujournal.net/Fourier.pdf

[2] More J.J , The Levenberg-Marquardt Algorithm: Implementation and Theory, Numerical Analysis, G.A.Watson (ed), Springer-Verlag, 1977